ODISHA 2012 Sunil kumar saini

36 $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$

$\Rightarrow x^2 + (a+b-2c)x + ab - c(a+b) = 0$

$\because$ sum of roots $-\frac{b}{a} = 0 = a+b-2c$

$a+b = 2c$

product of roots $= \frac{c}{a} = ab - c(a+b) = -(\cancel{c})$

$= -$

$\Rightarrow ab - c(a+b)$

$\Rightarrow ab - \frac{(a+b)^2}{2} \Rightarrow -\frac{(a^2+b^2)}{2}$ $\boxed{D}$

37

$x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$\Rightarrow \cancel{3-3(xyz)}$

$(x+y+z)^2 = 1^2 = x^2+y^2+z^2+2(xy+yz+zx)$

$= 2 + 2(xy+yz+zx) = 1$

$xy+yz+zx = -\frac{1}{2}$

$\therefore 3-3xyz = 1[2-(-\frac{1}{2})] = \frac{5}{2}$

$xyz = \frac{\cancel{5}}{2} = \frac{6-5}{2} = (\frac{1}{2})\frac{1}{3}$

$\boxed{xyz = \frac{1}{6}}$ $\boxed{B}$

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39 Let four No' be a, b, c, d and e

ACQT

$a+b+c = 180$, — (I)

$b+c+d = 197$ — (II)

$c+d+a = 208$ — (III)

$d+a+b = 222$ — (IV)

by solving $d-a = 17 \Rightarrow d = 17+a$

$a-b \Rightarrow 11 \Rightarrow b = a-11$

$b-c \Rightarrow 14 \Rightarrow c = b-14 \Rightarrow a-25$

$\therefore a+b+c+d = 268$

$a+a-11+a = 25+17+a = 269$

$a = 72$

largest No. $\boxed{d = a+17 = 89}$ $\boxed{C}$

42 $t_{11} = a+10d = 38$ — (I)

$t_{16} = a+15d = 73$ — (II)

By eq (I) & (II)

$a = -32$, $d = 7$

$\therefore t_{31} = -32+(30)\times 7 = \underline{178}$ $\boxed{A}$

43 $f(n+1) = f(n) + \frac{1}{2}$

or $f(n+1) - f(n) = \frac{1}{2}$

$\therefore$ it form an AP with common different

$= \frac{1}{2}$

$f(1) = 2$, $a_1 = 2$

$\therefore f(101) = a_{101} = a+100d = 2+100\times\frac{1}{2} = \underline{52}$ $\boxed{B}$

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AnicHA - 2013

44 Let the sides of the triangle with perimeter 8 unit be 3, 3, and 2

so area $= \frac{1}{2} \times 2 \times \sqrt{9^2-1^2} = 2\sqrt{2}$ unit^2 [B]

46 median of $\frac{x}{7}, \frac{x}{5}, \frac{x}{6}, x, \frac{x}{4}, \frac{x}{3}, \frac{x}{2}$ is 8

Arranging the no's

$\frac{x}{7}, \frac{x}{6}, \frac{x}{5}, \frac{x}{4}, \frac{x}{3}, \frac{x}{2}, x$

$\frac{x}{4} = 8 \qquad x = 32$ [e]

47 $m = 777 \text{ --- } 77$ [99 (items)]

$n = 999 \text{ ------ } 99$ [~~77~~ items 87]

$m \times n = 7 \times 10^{77} - 1$

$= 77 \text{---} \times 7 \times 10^{77} - 77 \text{---} 7$

$= 99 + 77$ digits

$= 77 \text{--} 769 \text{--} 92 \text{---} 33$

So sum of digits $= 76 \times 7 + 6 \times 22 \times 9 + 76 \times 2 + 3 = 891$ [B]

48

$\sin^2 1° + \sin^2 2° + \dots + \sin^2 89°$

$\sin^2 1° + \sin^2 2° + + \dots + \sin^2 45° + \dots + \cos^2 2° + \cos^2 1°$

$= (\sin^2 1° + \cos^2 1°) + \dots$

$\dots (\sin^2 44° + \cos^2 44°) + \sin^2 45° = 44 + \frac{1}{2} = 44\frac{1}{2}$ [C]

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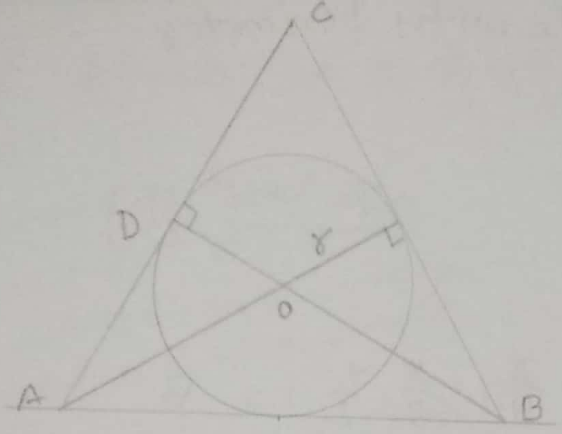
50



Area of circle = $\pi r^2 = 48\pi$

$r = \cancel{\frac{48}{\sqrt{3}}}\ 4\sqrt{3}$

$BD = 3r = 12\sqrt{3}$

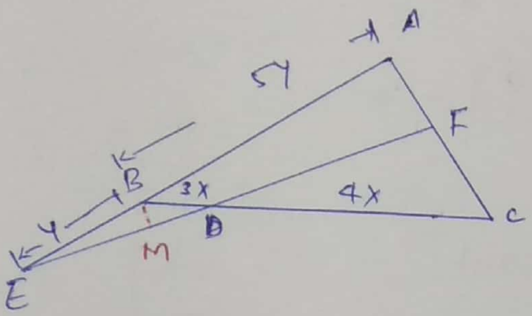
$BD = \frac{\sqrt{3}}{2} AB$

$12\sqrt{3} = \frac{\sqrt{3}}{2} AB$

$AB = 24$

Perimeter $= 3 \times 24 = \underline{72}$ $\boxed{D}$

54



Construction :-

Draw $BM \parallel AE$

so $\triangle BME \sim \triangle CFD$

or $CF = \frac{4BM}{3}$ --- (1)

Now $\triangle AEF \sim \triangle BEM$

$\therefore \frac{BM}{AF} = \frac{BE}{AE} = \frac{6}{6y}$

$AF = BM \times 6$ --- (2)

$\frac{CF}{AF} = \underline{\frac{2}{9}}$ $\boxed{A}$

55

$V_1 = \frac{1}{3}\pi r^2 h =$

$V_2 = \frac{1}{3}\pi \left(\frac{120r}{100}\right)^2 \times \frac{120h}{100} = \frac{1728}{1000}\left(\frac{1}{3}\pi r^2 h\right)$

$V_2 - V_1 = \frac{728}{1000}\left(\frac{1}{3}\pi r^2 h\right)$

% increase $= 72.8\%$ $\boxed{D}$

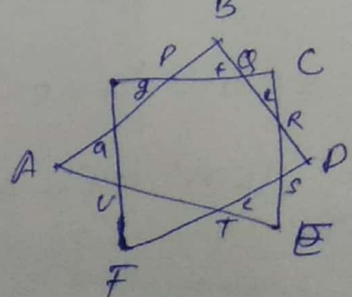
53 PQRSTUV is a heptagon so

Sum of all exterior angles $= 360^\circ$

Now there are 7 triangles $= 7 \times 180 = 1260$

$a+b+c+d+e+p+y = 360^\circ$

$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle 260 - 2 \times 36 = \underline{540}$ $\boxed{D}$



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36 Mean = $\bar{x}$ = if each core is divided by α

New mean = $\frac{\bar{x}}{\alpha}$, increased by 10

$\therefore$ New mean = $\frac{\bar{x}}{\alpha} + 10 = \frac{\bar{x} + 10\alpha}{\alpha}$ (C)

37 $\tan 15 \tan 25 \tan 60^\circ \tan 65 \tan 75^\circ$

$\Rightarrow \tan(90^\circ - 75) \tan(90 - 65) \cdot \tan 60^\circ \tan 65^\circ \tan 75^\circ$

$\Rightarrow \cancel{\cot 75^\circ} \cancel{\cot 65} \cdot \tan 60^\circ \cancel{\tan 65} \cdot \cancel{\tan 75^\circ}$

$\Rightarrow \tan 60^\circ = \sqrt{3}$ (A)

38 $\sec\theta + \tan\theta = x$ — (i)

$\therefore \sec\theta - \tan\theta = \frac{1}{x}$ — (ii)

$\boxed{1 + \tan^2\theta = \sec^2\theta \quad 1 = \sec^2\theta - \tan^2\theta}$

eq (ii) - (i)

$2\tan\theta = \frac{x^2 - 1}{x} \Rightarrow \tan\theta = \frac{x^2 - 1}{2x}$ (D)

39 $f(x) = x^3 + 5x^2 + Kx$

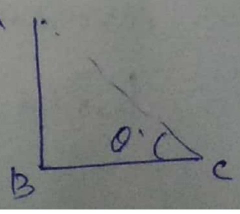
if $(x+3)$ divides $f(x)$

$x = -3$

$\therefore f(-3) = (-3)^3 + 5(-3)^2 + K(-3) = 0$

$\Rightarrow -27 + 45 - 3K$ $\boxed{K = 6}$ (C)

40 let AB is pillar, BC it shadow



$\frac{AB}{BC} = \frac{1}{\sqrt{3}} = \tan\theta = \tan 30^\circ \quad \theta = 30^\circ$ (D)

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42 $a^x = b$, $b^y = c$, $c^z = a$

$\Rightarrow b^y = c$

$\Rightarrow (a^x)^y = c$ $\qquad$ $a^{xy} = c$

if $c^z = a$ $\Rightarrow$ $a^{xyz} = a^1$

$xyz = 1$

$\boxed{x^2y^2z^2 = 1}$ $\qquad$ (B)

44 $an^2 + b = 0$

$\Rightarrow D^2 > 0$

$\Rightarrow B^2 - 4AC > 0$

$\Rightarrow -4AC > 0$

$ab < 0$ $\qquad$ (D)

45 $\alpha n^2 + \beta x + \gamma = 0$ —— (1)

sum of roots $= -\frac{\beta}{\alpha} = x_1 + x_2 = 3$

$-3\alpha = \beta$

product of roots $= \frac{\gamma}{\alpha}$ $\qquad$ $\gamma = 2\alpha$

putting value's of β, γ in (1)

$-3n^2\alpha + \alpha n + 2\alpha = -3n^2 + n + 2 = 0$

$x^2 - x - 2 = 0 = (x-2)(x+1)$

$\boxed{x = -1, 2}$ $\qquad$ (D)

46 Total no. of event $2^4 = 16$

favourable events ⟨HHTT, HTHT, HTTH, THTH, TTHH, THTH⟩

Probability $= \frac{6}{16} = \frac{3}{8}$ $\qquad$ (B)

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47 let radius $r = r$
height $h = h$
Volume $= \pi r^2 h$
New radius $r' = \frac{r}{2} = \frac{r}{2}\ \frac{50}{100} r = \frac{1}{2} r$
New height $h' = \frac{150}{100} h = \frac{3}{2} h$
New Volume $V' = \pi r'^2 h'$

$$V' = \pi \left(\frac{r}{2}\right)^2 \frac{3}{2} h = \frac{3}{8} \pi r^2 h$$

$V' = 0.375 V = 37.5 V\%$
decreased Volume $= \underline{62.5\%}$ (C)

48 Volume of sphere = T.S.A of sphere
$\frac{4}{3}\pi r^3 = 4\pi r^2$
$\boxed{r = 3}$ (B)

49 volume of larger cone $= \frac{1}{3}\pi r^2 h$
volume of smaller cone $= \frac{1}{3}\pi r^2 H$
A.T.Q
$= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 H \Rightarrow \underline{\frac{H}{h} = \frac{1}{3}}$
(B)

51 $A(2,3)\ B(-1,2)\ C(x,4)$
Divide the line AB in $3:4$
$x = \frac{3(-1)+4(2)}{3+4}$, $y = \frac{3\times 2+12}{3+4}$.
$x = \frac{5}{7}$ $y = \frac{18}{7}$
C $\left(\frac{5}{7}, \frac{18}{7}\right)$ lie on line $x + 2y = K$
$\frac{5}{7} + \frac{18\times 2}{7} = K \Rightarrow K = \frac{41}{7}$ (A)

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Q 52 $A(1,1),\ B(-1,1),\ C(-\sqrt{3}, K)$

$AC = BC = \sqrt{(-\sqrt{3}-1)^2+(K-1)^2} = \sqrt{(-\sqrt{3}+1)^2+(K+1)^2}$

$= 3+1+2\sqrt{3}+K^2+1-2K = 3+1-2\sqrt{3}+K^2+1+2K$

$= \cancel{3+1}\ 4\sqrt{3} = 4K$

$K = \sqrt{3}$ (B)

54 let $AB = x$

then $PT^2 = PA \times PB$

$12^2 = (\cancel{x}+8)\,8$

$\frac{144}{8} = 18 = (\cancel{x}+8)$

$x = AB = 10$ cm (C)

55 EDC is equilateral triangle

$\angle ECD = 60°$ and $EC = CD$

ABCD is square

$\angle BCD = 90°$ and $BC = CD$

$\Rightarrow EC = CD = BC$

$\Rightarrow EC = BC$

In $\triangle BCE$

$BC = CE$

$\therefore \angle CEB = \angle CBE$

$2\angle BCE = \angle BCD + \angle ECD$

$\angle BCE = 90+60 = 150$

$\angle BCE + \angle CEB + \angle CBE = 180°$

$2\angle ECB = 30°$

$\angle ECB = 15°$ (A)

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36

Volume of Rod = Volume of wire

$$\left(\frac{\pi}{4} d^2 . h\right)_{Rod} = \left(\frac{\pi}{4} d^2 h\right)_{wire}$$

$$\Rightarrow (d^2 . h)_{Rod} = (d^2 . h)_{wire}$$

Rod	wire
d = 1cm	d = ?
h = 8cm	h = 18m = 1800cm

$$\Rightarrow 1^2 \times 8 = d^2 \times 1800$$

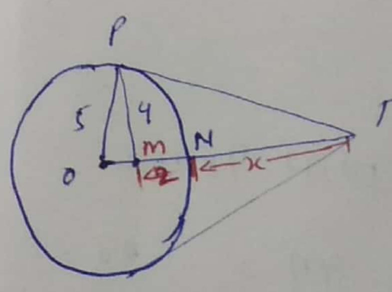
$$d^2 = \frac{8}{1800} = \frac{4}{900}$$

$$d = \frac{2}{30} \text{ cm}$$

$$\boxed{\text{thickness} = \frac{d}{2} = \frac{1}{30} \text{ cm}}$$

$\boxed{B}$

37

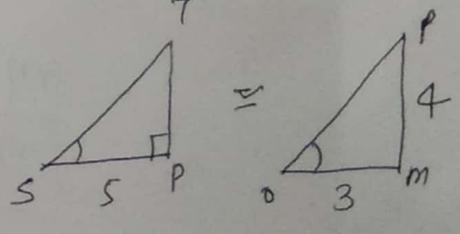


∵ OP = 5

PM = 4

MN = 2

OM = 3 = $\sqrt{5^2 - 4^2}$



Let NT = x

∴ In Triangle POT ≅ △PMO

$$\frac{PT}{5} = \frac{4}{3}$$

$\boxed{PT = \frac{20}{3} \text{ cm}}$ $\boxed{C}$

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38

$\sqrt{8+2\sqrt{8+2\sqrt{8+2\sqrt{8}}}} = x$ (let)

$x^2 = 8 + 2\underbrace{\sqrt{8+2\sqrt{8+2\sqrt{8}}}}_{x}$

$x^2 = 8 + 2x$

$\therefore x^2 - 2n - 8 = 0$

$n^2 - 4n + 2n - 8 = 0$

$\Rightarrow (n-4)(n+2) = 0$

$x = -2$ ~~not sol~~ ✗ Not possible

$\boxed{x = 4}$ (A)

39

Sum of deviation from 50 = $(x_1 - 50) + (x_2 - 50) + \cdots + (x_n - 50) = -10$

Sum of deviation from 46 = $(x_1 - 46) + (x_2 - 46) + \cdots + (x_n - 46) = 70$

$\therefore (x_1 + x_2 + \cdots x_n) - 50n = -10$ — (I)

$(x_1 + x_2 \cdots x_n) - 46n = 70$ — (II)

eq (I) − (II)

$-50n + 46n = -80$

$4n = 80$

$\boxed{n = 20}$

mean, $50 - \frac{10}{20} = \underline{49.5}$ (C)

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40

$\sqrt{2}, \sqrt[3]{4}, \sqrt[4]{6}$

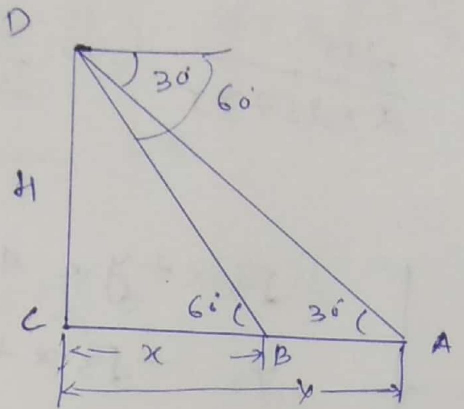
L.C.M = 12

$\Rightarrow \sqrt[12]{2^6}, \sqrt[12]{4^3}, \sqrt[12]{6^3}$

$\sqrt[12]{64}, \sqrt[12]{256}, \sqrt[12]{216}$

Ascending order $\sqrt{2} < \sqrt[4]{6} < \sqrt[3]{4}$ $\boxed{B}$

41



Δ ACD

$\therefore \tan 30° = \frac{H}{y} \Rightarrow H = \frac{y}{\sqrt{3}}$ — (I)

Δ BCD

$\therefore \tan 60° = \frac{H}{x} \Rightarrow H = x\sqrt{3}$ — (II)

(I) = (II)

$\frac{y}{\sqrt{3}} = x\sqrt{3} \Rightarrow \boxed{y = 3x}$

$AC = 3x$

$AB = 3x - x = \underline{2x}$

2x distance covering time = 12 sec

x " " " 6 sec

3x " " " = 18 sec

$\therefore$ CB $\Rightarrow$ x distance = 6 sec $\boxed{A}$

42

$S = \frac{1}{2\times7} + \frac{1}{7\times12} + \frac{1}{12\times17} + \cdots \frac{1}{252\times257}$

$\Rightarrow \frac{1}{5}\left(\frac{1}{2}-\frac{1}{7}\right) + \frac{1}{5}\left(\frac{1}{7}-\frac{1}{12}\right) + \cdots \frac{1}{5}\left(\frac{1}{252}-\frac{1}{257}\right)$

$\Rightarrow \frac{1}{5}\left[\frac{1}{2} - \cancel{\frac{1}{7}} + \cancel{\frac{1}{7}} - \cancel{\frac{1}{12}} + \cancel{1} - \cdots - \cancel{\frac{1}{252}} + \cancel{\frac{1}{252}} - \frac{1}{257}\right]$

$\Rightarrow \frac{1}{5}\left(\frac{1}{2} - \frac{1}{257}\right) = \frac{255}{2\times257\times5} = \frac{31}{2\times\cancel{25}7}$ $\boxed{D}$

43

$x + \sqrt{3}y = 4$

$y = -\frac{x}{\sqrt{3}} + \frac{4}{\sqrt{3}}$

$m_1 = -\frac{1}{\sqrt{3}}$

$\sqrt{3}x + y = 4$

$y = -\sqrt{3}x + 4$

$m_2 = -\sqrt{3}$

or $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \left[\frac{1}{\sqrt{3}} \neq \frac{\sqrt{3}}{1}\right] = \frac{4}{4}$

distance From $(0, 0) = \frac{4}{\sqrt{1^2+(\sqrt{3})^2}} = \frac{4}{4} = 1$

And $\frac{4}{\sqrt{(\sqrt{3})^2+1^2}} = \frac{4}{4} = 1$

at equi-distance D

44

$r = 8$ cm $H = 3$ cm

$\because$ Volume of cylinder $= \pi r^2 h$

$\pi r^2 h = \pi \times 8^3 \times 3 = 603.186$

for keep Volume same $r^2 h$ should be same.

if $\left(r + \frac{16}{3}\right)$. or $\left(h + \frac{16}{3}\right)$ we get same

Volume i.e

$r = r + \frac{16}{3} = 8 + \frac{16}{3} = \frac{40}{3}$ $\because$ $r^2 h = \frac{1600}{3}$

or

$H = H + \frac{16}{3} = 3 + \frac{16}{3} = \frac{25}{3} \Rightarrow r^2 h = \frac{1600}{3}$ $\boxed{B}$

45     $lx^2 + nx + n = 0$ let roots p, q $\because$

$p + q = -\frac{n}{l}$ — (i) , $pq = +\frac{n}{l}$ (ii)

$p + q + pq = 0$ divided by $\sqrt{pq}$

$\because \frac{p}{\sqrt{pq}} + \frac{q}{\sqrt{pq}} + \frac{pq}{\sqrt{pq}} = 0$

$\Rightarrow \sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$ $\boxed{C}$

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46 line passing through (1, 3), (2, 7) is

$$y-3 = \frac{7-3}{2-1}(x-1)$$

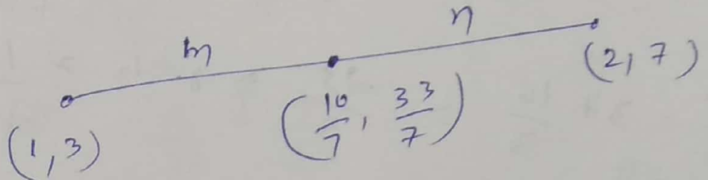
$$y-3 = 4x-4$$

$$4x-y-1=0 \quad \text{—(I)}$$

$$3x+y-9=0 \quad \text{—(II)}$$

By eq (I) & (II)

$$x = \frac{10}{7}, \quad y = \frac{33}{7}$$



$$2m+n = \frac{10}{7}$$

$$7m+2n = \frac{33}{7}$$

$$m = 3/7, \quad n = \frac{4}{7}$$

$$\boxed{m : n = 3 : 4 \quad \text{(D)}}$$

47 $\sqrt{999 \times 1000 \times 1001 \times 1002 + 1}$

$$\sqrt{1 + 1.00199 \times 10^{12}} = \sqrt{1.001999 \times 10^{12}}$$

$$= \underline{1000999} \quad \text{(C)}$$

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48

Probability:

Total chance = 36

total Out comm for 8 = (3,5)(5,3),(6,2)(2,6),(4,4)

= 5

$P(8) = \frac{5}{36}$ $\boxed{D}$

49

$\because$ $\tan x \times \tan(60+x) \times \tan(60-x) = \tan 3x$

So $\because$ $\tan 18 \cdot \tan(60+18) \times \tan(60-18) = \tan 3\times 18$

$= \tan 54$

$\tan 42° \tan 78° = \frac{\tan 54}{\tan 18}$ — (I)

Now $\tan 6 \cdot \tan(60-6) \tan(60+6) = \tan 3 \cdot 6$

$= \tan 18$

$\tan 66 \cdot \tan \cancel{54} = \frac{\tan 18}{\tan 54}$ — (II)

BY multipling (I) & (II)

$\tan 6 \tan 42 \tan \cancel{66} \tan 78 = 1$ $\boxed{C}$

50 $\operatorname{cosec} 10^\circ - \sqrt{3} \sec 10^\circ$

$\Rightarrow \frac{1}{\sin 10} - \frac{\sqrt{3}}{\cos 10}$

$\Rightarrow \frac{(\cos 10^\circ - \sqrt{3}\sin 10)}{\sin 10^\circ \cos 10^\circ} \Rightarrow \frac{\frac{1}{2}[2\cos 10^\circ - 2\sqrt{3}\sin 10^\circ]}{\frac{1}{2}[2\sin 10 \cos 10]}$

$\Rightarrow \frac{2[\cos 10 \cdot \sin 30 - \sin 10 \cdot \cos 30^\circ]}{\frac{1}{2} \sin 20}$

$\Rightarrow 4 \frac{\sin(30-10)}{\sin 20} = 4$ $\boxed{A}$

51 SP = 12000/-

CP of motor cycle

$\because \frac{SP}{CP} = \frac{80}{100}$

$SP = \frac{12000 \times 100}{80} = 15000$

loss = 3000 /-

Scooter SP of scooter

$CP = \frac{12000}{120} \times 1000 = 10,000$

profit = 2000

net = $-3000 + 2000 = \underline{-1000}$ $\boxed{B}$

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52 Let x part invest at 4%.

remaining part $(4500-x)$ at 6%.

SI $\Rightarrow x \times \frac{4}{100} \times \cancel{t} = (4500-x) \times \frac{6}{100} \times \cancel{t}$

$\Rightarrow 10x = 4500 \times 6$

$\boxed{x = 2700}$

$\therefore 4500 \times \frac{r}{100} \times \cancel{t} = \left[\left(2700 \times \frac{4}{100}\right) + \left(\frac{1800 \times 6}{100}\right)\right] \cancel{t}$

$45x = 108 + 108 = 216$

$\boxed{r = 4.8\%}$ (B)

53 Let still water speed is $= x = 18$ km/hr

speed of stream $= y$ km/hr

effective speed in up stream $= x - y$

" in down stream $= x + y$

"

$\therefore$ [time taken up stream] − [time taken down stream] = 1

$\Rightarrow \frac{\text{Distance}}{x-y} - \frac{\text{Distance}}{x+y} = 1$

$\frac{24}{x-y} - \frac{24}{x+y} = 1$

$\boxed{x = 18, \quad y = 6 \text{ km/hr}}$ (B)

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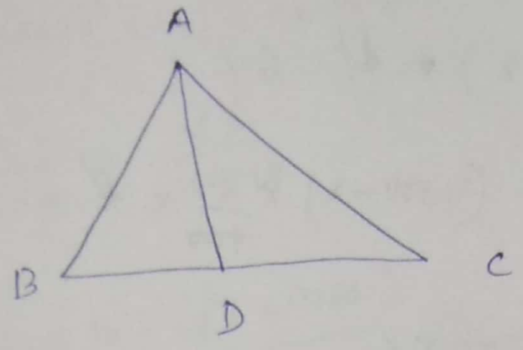
54



$\because \angle B = \angle DAB$

$\because AD = BD$

$\because$ By construction

$\angle ADC = \angle DAB + \angle B$ (Ext. angle sum interior property)

$= 2\angle B$

$\angle CAD = \angle CAB - \angle DAB = \angle B$

given that $\angle A = 2 \cdot \angle B$

$\because \triangle ACD \sim \triangle BCA$

$\because$ According similar triangles

$$\frac{AC}{BC} = \frac{AD}{AB}, \quad \frac{AC}{BC} = \frac{CD}{AC}$$

$\therefore AC \times AB = BC \times AD, \quad AC^2 = BC \times CD$

$\because BC^2 = BC \times (BD + CD) = BC \times (AD + CD)$

$= AC \times AB + AC^2$

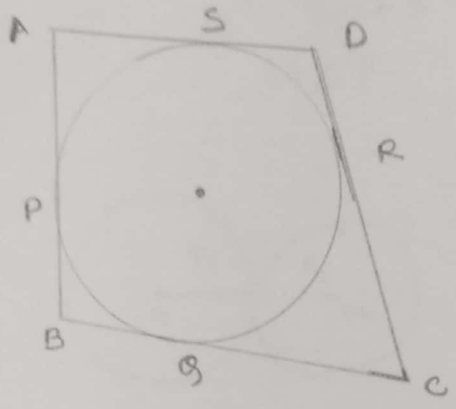
$BC^2 = AC[AB + AC]$ $\boxed{(B)}$

55

BC = 38 cm

CD = 25 cm

BP = 27 cm



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ODISHA - 2015

41 To have real roots $D \geq 0$ $(b^2 - 4ac)$

$$x^2 + kx + 64 = 0$$

$$k^2 - 4(64) \geq 0$$

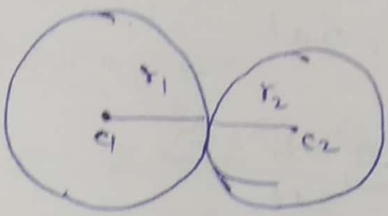
$$(k-16)(k+16) \geq 0$$

$$k \in (-\infty, -16] \cup [16, \infty)$$

$$x^2 - 8x + k = 0$$

$$16 \geq k \qquad \boxed{k = 16} \quad (B)$$

42



$$\pi r_1^2 + \pi r_2^2 = 130\pi$$

$$r_1^2 + r_2^2 = 130 \quad \text{---} (i)$$

$$c_1c_2 = r_1 + r_2 = 14 \quad \text{---} (ii)$$

$$r_1 = 11, \quad r_2 = 3 \qquad (c)$$

43 $$\frac{\cos^2\theta - 3\cos\theta + 2}{\sin^2\theta} = 1$$

$$\cos^2\theta - 3\cos\theta + 2 = \sin^2\theta$$

$$2\cos^2\theta - 3\cos\theta + 1 = 0$$

$$\cos\theta = 1, \quad \cos\theta = \frac{1}{2}$$

$$\theta = 90^\circ \quad \theta = \pi/3 = 60^\circ \quad (B)$$

$$0 < \theta < \frac{\pi}{2}$$

44. $$\frac{2+3+5+7+9+11+13+17+19+23+29}{10} = \frac{129}{10} = 12.9 \quad (c)$$

45 $\Delta ABC \sim \Delta DEF$

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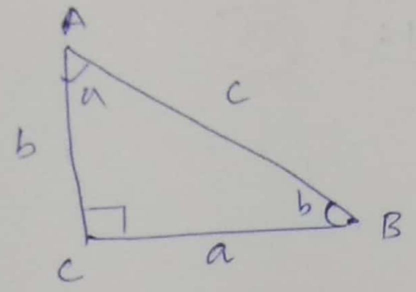
$$\frac{[ABC]}{DEF} = \frac{BC^2}{EF^2}$$

$$\frac{243}{108} = \frac{6^2}{EF} \Rightarrow \frac{9}{6} = \frac{6}{EF}$$

$$EF = \frac{36}{9} = \underline{4} \quad (D)$$

46



$AB = AC \times BC$

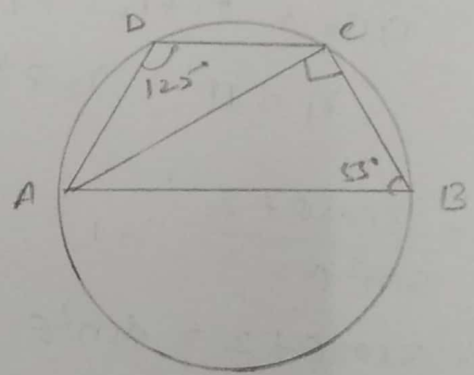
$c^2 = 2ab$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$0 = a^2+b^2-2ab$

$A-b=0, \quad \underline{a=b} = \underline{45^\circ}$ (C)

47 AB is diameter



$\angle ACB = 90°$

$\because \angle ABCD$ is cyclic

$\angle D + \angle B = 180°$

$\angle B = 55°$

in $\triangle ABC$ $\angle B = 55°$ $\angle C = 90°$ $\angle A = \underline{35°}$ (A)

49 $\sin\theta + \operatorname{cosec}\theta = 2$

$$\sin\theta + \frac{1}{\sin\theta} = 2$$

$\sin^2\theta + 1 = 2\sin\theta$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$

$\sin\theta = 1 \qquad \operatorname{cosec}\theta = 1$

$\sin^{13} + \operatorname{cosec}^{13} = 1+1 = \underline{2}$ (D)

50 Given $abc = 196$

$R = 2.5$

$\because R = \frac{abc}{4A}$

where Δ = Area of triangle

$\Delta = \frac{abc}{4R} = \frac{196}{4 \times 2.5} = 19.6$ (B)

51 $l+b+h = 19$

d = diagonal = $\sqrt{l^2+b^2+h^2} = 5\sqrt{5}$

$l^2+b^2+h^2 = 125$

Surface $= 2(lb+lh+bh)$

$\Rightarrow (l+b+h)^2 - (l^2+b^2+h^2) = 361 - 125 = \underline{236}$ (B)

52 $pqr = 1$

$\frac{1}{1+p+\frac{1}{q}} + \frac{1}{1+p+pq} + \frac{1}{1+\frac{1}{pq}+\frac{1}{p}}$

$= \frac{q}{pq+q+1} + \frac{1}{pq+q+1} + \frac{pq}{pq+q+1} = \frac{pq+q+1}{pq+q+1} = 1$ (C)

53 Two lines are perpendicular if product of slopes is $-ve$

i.e $a_1a_2 + b_1b_1 = 0$ $m_1 = \frac{2}{3}$, $m_2 = -3/2$ (B)

$m_1 m_2 = -1$

56 Given E = all dice show differenance face

$P(E) = 1 \times \frac{5}{6} \times \frac{4}{6} = \frac{5}{9}$ (D)

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57

$T_4 = 11, \ T_{10} = 16$

$a + 3d = 11, \quad a + 9d = 16 = 5d$

$\Rightarrow d \Rightarrow 5/6$

$a = 11 - 3d = 11 - 3\left(\frac{5}{6}\right)$

$\Rightarrow 11 - \frac{5}{2} = \frac{17}{2}$

$S_{40} = \frac{40}{2}\left[2 \times \frac{17}{2} + 39\left(\frac{5}{6}\right)\right] = 20\left(17 + \frac{13 \times 5}{2}\right)$

$= 10(34 + 65) = 99 \times 10 = \underline{\underline{990}}$ (D)

58 $A(2,1)$, $B(x,y)$ $C(7,5)$ are collinear slope of AB = slope BC

$\Rightarrow \frac{y-1}{x-2} = \frac{5-y}{7-x} \Rightarrow 7y - xy - 7 + x$

$\Rightarrow 5x - xy - 10 + 2y$

$4x - 5y - 3 = 0$ (D)

60 Let speed of boat = x km/hr

Speed of stream = y km/hr

Speed still water = $(x+y)$ km/hr

speed of up stream = $(x-y)$ km/hr

$t = \frac{d}{x+y} + \frac{d}{x-y}$

$4\frac{1}{2} = \frac{30}{x+y} + \frac{30}{x-y} \Rightarrow \frac{30}{15+y} + \frac{30}{15-y} = \frac{9}{2}$

$\boxed{y = 5 \text{ km/hr}}$ (B)

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56 $a + 8b = 14$ —(i)

$5a - 2b = 16$ (ii)

eq $4 \times$ (ii)

$20a - 8b = 64$ —(iii)

eq (i) + (iii)

$a = \frac{26}{7}$, $b = \frac{72}{7 \times 8} = \frac{9}{8}$

$\frac{a+b}{2} = \left[\left(\frac{1}{7} \times 26\right) + \frac{9}{8}\right]\frac{1}{2} = 2.5$ $\boxed{D}$

57 Vowel in word → A E I = 3

total letters are = 11

$p(\text{vowel}) = \frac{3}{11}$ $\boxed{c}$

58 line Joiny $(c, 8)$ & $(a, 0)$

$\therefore$ $y - 8 = \frac{8}{a-c}(x - c)$

$y = \frac{8}{a-c}(x-c) + 8$ $m_1 = \frac{8}{a-c}$

line Joiny $(-c, c)$, $(3c, a)$

$y = \frac{a-c}{4c}(x + c) + c$ $m_2 = \frac{a-c}{4c}$

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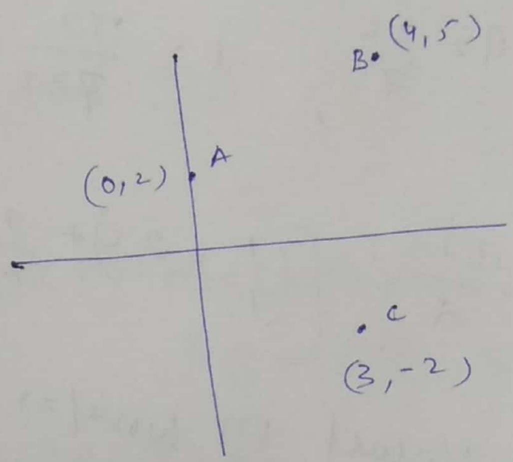
$m_1 \times m_2 = -1$

$\frac{8}{a-c} \times \frac{a-c}{4c} = -1$

$\boxed{c = -2}$

$\boxed{a + c = 10 - 2 = 8}$

58



$AB = \sqrt{(4-0)^2 + (5-2)^2} = 5$

$AC = \sqrt{(3-0)^2 + (2+2)^2} = 5$

$AB = AC$ option $(0,2)$ $\boxed{A}$

59 $A+B+C = 180°$, $\cos B \cos C = \cos A$

$A = 180 - (B+C) \Rightarrow \cos B \cos C = \cos[180 - (B+C)]$

$= -\cos(B+C)$

$\Rightarrow \cos B \cos C = -\cos B \cos C + \sin B \sin C$ By cross multiple

$\Rightarrow 1 = -1 + \tan B \tan C$ By transfer

$\boxed{\tan B \cdot \tan C = 2}$ $\boxed{C}$

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60

$3 \cdot 5^{2x-1} - 2 \times 5^{x-1} = 0.2$

$\Rightarrow \frac{3 \cdot 5^{2x} - 2 \cdot 5^{x}}{\cancel{5}} = \frac{1}{\cancel{5}}$

$\Rightarrow 3 \cdot 5^{2x} - 2 \cdot 5^{x} = 1$

Now By option

If $x = 0$ then $3 - 2 = 1$ equation is saitsfy

$\therefore x = 0$, $\boxed{D}$

61 $4x^2 - 20x = p^2$ roots are α And β.

$\Rightarrow 4x^2 - 20x - p^2 = 0$

Sum of roots $\alpha + \beta = 5$

product of roots $\alpha\beta = \frac{-p^2}{4}$

$\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4 \cdot \alpha\beta}$

$\Rightarrow \sqrt{25 + 4 \cdot \frac{p^2}{4}}$

$\alpha - \beta \Rightarrow \cancel{\sqrt{25+p^2}}$.

$\sqrt{25 + p^2}$ $\boxed{A}$

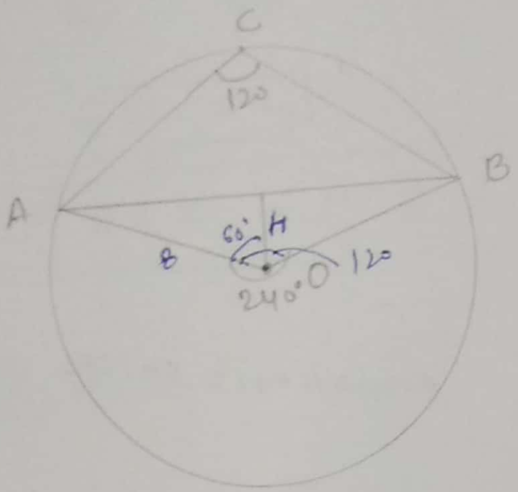
62 $2\angle A + 2\angle B = 3\angle B$

$\boxed{2\cdot\angle A = B}$, $\boxed{3B = \angle C}$

$A + B + C = 180$

$\frac{B}{2} + B + 3B = 180$

$\Rightarrow B = 40^\circ, \ A = 20^\circ, \ C = 120^\circ$



Dia = 16

Radius = AO = OB = 8

ΔAOB is iso-sceles tringle

$AB = 2 \times 8 \sin 60^\circ = 8\sqrt{3}$ cm

$H = 8\cos 60^\circ = 4$ cm

Area $= \frac{1}{2} AB \times H = \frac{1}{2} \times 4 \times 8\sqrt{3}$

$= 16\sqrt{3}$ cm^2 (D)

63 $a+b = 3, \quad ab = 2, \quad 2^{a^3-b^3}$

$\because a^3 - b^3 = (a-b)(a^2+b^2+ab)$

$\because a-b = \sqrt{9-8} = 1$

$a^2+b^2+2ab = (a+b)^2 = 9$

$a^2+b^2 = 9 - 2.2 = 5$

$\Rightarrow a^3 - b^3 = 1(5+2) = 7$

$2^7 = 128$ $\boxed{C}$

64 $x^3 - 6x^2 - x + 30$

$\because 5(\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{c}{a} = -\frac{1}{1} \times 5 = -5$ $\boxed{B}$

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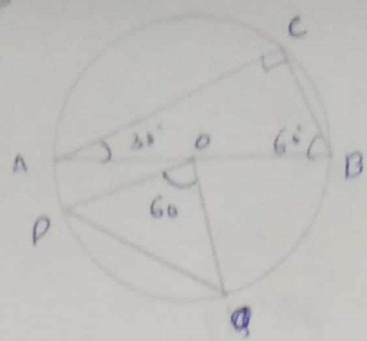
65



$AO = OB = PO = OQ = r$

$BC = 2r\cos 60° = r$

$AC = 2r \cdot \sin 60° = r\sqrt{3}$

ΔABC area $= \frac{1}{2} r^2 \sqrt{3} = \frac{\sqrt{3}}{2} r^2$

ΔPOQ is equilateral triangle.

Area is $\frac{\sqrt{3}}{4} r^2$

$\therefore \frac{\text{Area } \Delta POQ}{\text{Area } \Delta ABC} = \frac{\sqrt{3}/4 r^2}{\sqrt{3}/2 r^2} = \frac{1}{2}$ $\boxed{D}$

66 $x^2 - 4x - \log_3 a = 0$

Root of equation are real, so.

Discriminant $= 0$ for real Roots

$b^2 - 4ac = 0$

$(-4)^2 + 4\log_3 a = 0$

$\log_3 a = -4$

$\boxed{a = 3^{-4} = \frac{1}{81}}$ $\boxed{B}$

67 Sum of 12 Sides $= 6$ cm

1 side $= \frac{1}{2}$ cm

Volume of cube is $= (\text{Side})^3 = \left(\frac{1}{2}\right)^2 = \frac{1}{8}$ cm^3. $\boxed{A}$

68 $\operatorname{cosec}\theta + \cot\theta = m$. —(1)

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\Rightarrow \operatorname{cosec}^2\theta - \cot^2\theta = 1$

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6. $\operatorname{cosec}\theta - \cot\theta = \frac{1}{m}$ — (II)

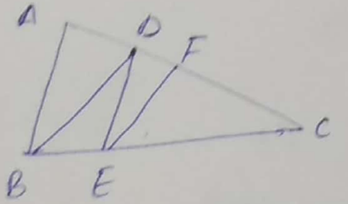
Adding eq (I) + (II)

$2\operatorname{cosec}\theta = \frac{1+m^2}{m}$

$\sin\theta = \frac{2m}{1+m^2}$, $\cos\theta = \frac{m^2-1}{m^2+1}$

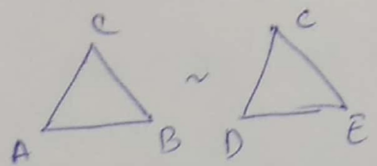
$\boxed{\sec\theta = \frac{m^2+1}{m^2-1}}$ $\boxed{D}$

69



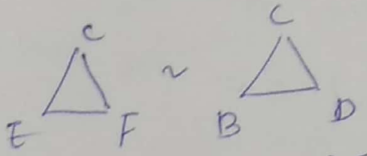
$\overline{DE} \parallel \overline{AB}$, $\overline{EF} \parallel \overline{BD}$

$CF = 4cm$, $AC = 9cm$



∴ $\triangle CAB \sim \triangle CDE$

$\frac{AC}{DC} = \frac{BC}{CE} = \frac{AB}{DE}$



$\triangle CEF \sim \triangle CBD$

$\frac{CE}{BC} = \frac{CF}{CD} = \frac{EF}{BD}$

$\frac{DC}{AC} = \frac{CF}{CD} \Rightarrow CD^2 = 9\times 4$

$CD = \sqrt{36}$

$\boxed{CD = 6cm}$ $\boxed{B}$

70

$a:b = 3:5$, $a:c = 5:7$

$\frac{a}{b} = \frac{3}{5} = \frac{15}{25}$ $\frac{a}{c} = \frac{5}{7} = \frac{15}{21}$

$\frac{b}{c} = \frac{b}{a}\times\frac{a}{c} = \frac{15}{21}\times\frac{21}{15} = \frac{15}{25}$

$\Rightarrow \frac{25}{15}\times\frac{15}{21} = \frac{25}{21}$

$\frac{b}{c} = \frac{25}{21}$

$\frac{b+c}{c} = \frac{46}{21}$ $\frac{b-c}{c} = \frac{4}{21}$

$\boxed{\frac{b-c}{b+c} = \frac{4}{46}}$ $\boxed{A}$

71

$a_m = \frac{1}{n} = a + (m-1)d$ --- (I)

$\Rightarrow an + nmd - nd = 1$

$a_n = \frac{1}{m} = a + (n-1)d$ — (II)

$\Rightarrow am + mnd - md = 1$

By eq (I) & (II)

$an + mnd - nd = am + nmd - md$

$a(n-m) - (n-m)d = 0$

$a(n-m) = (n-m)d$

$\boxed{a = d}$

By eq (I) $an + mnd - nd = 1$

$d\cancel{n} + nmd - n\cancel{d} = 1$

$mnd = 1$

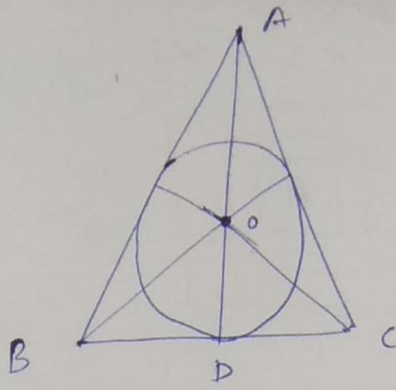
$mn = \frac{1}{d}$ ∴ $\boxed{a = \frac{1}{mn}}$

mn^{th} term $= a + (mn-1)d$

$\Rightarrow \frac{1}{mn} + (mn-1)\frac{1}{mn} = \boxed{1 \quad A}$

6 $\underline{\underline{72}}$

Let r is radius of circle then area $= \pi r^2$



$\therefore \pi r^2 = 48\pi$

$r^2 = 48$ cm

$\because$ incenter of a circle is the point of intersection of the ~~angular~~ angular bi-sectors

Given ABC is a equilateral triangle and AD = h be the altitude.

Hence these bisectors are also the altitude and medians whose point of intersection divides the median in the ratio 2:1

median in the ratio 2:1

$\angle ADB = 90^\circ$ and $OD = \frac{1}{3} AD$

$\therefore r = \frac{h}{3} \Rightarrow h = 3 \cdot r = 3 \times \sqrt{48}$

Let each side of triangle is a.

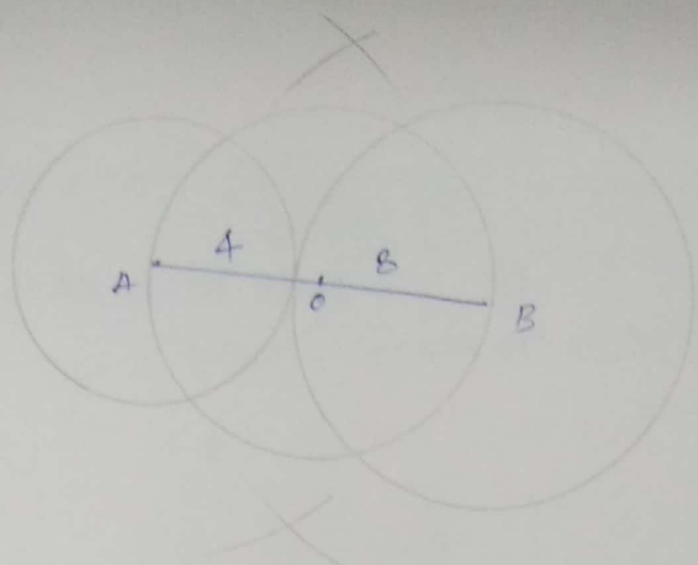
$\therefore$ Altitude $h = \frac{\sqrt{3}}{2} a$

$a = \frac{2h}{\sqrt{3}} = \frac{6\sqrt{48}}{\sqrt{3}} = 6\sqrt{\frac{48}{3}} = 24$ cm

~~Area = $\frac{\sqrt{3}}{4} \times a^2 =$~~

perimeter $= 3 \times 24 =$ 72 cm (D)

73

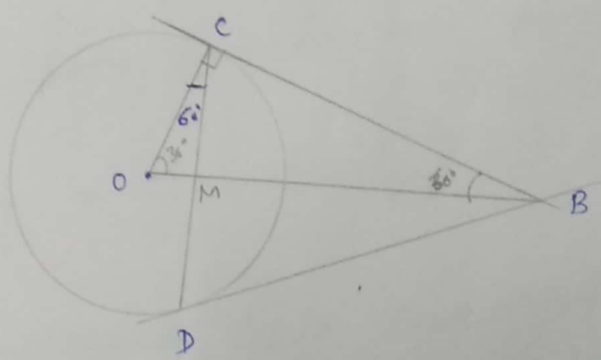


New circle with d/a AB = 12 cm.

radius = 6 cm

Area = $\pi r^2 = \underline{36\pi}$ $\boxed{B}$

74



OB = 12

$\angle CBO = 120°$

CD = ?

$OC = 12 \cos 30 = 6\sqrt{3}$ cm

$MCB = OC \cos 60° = 6\sqrt{3} \times \frac{1}{2} = 3\sqrt{3}$ cm

$CD = 2 \times CM = \underline{6\sqrt{3}}$ cm. (A)

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1 $\cos\theta + \sqrt{3}\sin\theta = \sqrt{2}$ $\quad 0 < \theta < 180$

$\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta = \frac{\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}$

$\sin 30^\circ \cos\theta + \cos 30^\circ \sin\theta = \frac{1}{\sqrt{2}}$

$\sin(\theta + 30^\circ) = \sin 45^\circ$ or $\sin 135^\circ$

$\theta = 45 - 30^\circ$ or $135 - 30$

$\boxed{\theta = 15^\circ}$ or 105°

(A)

2 $f(x)$ $\sin x + \cos x$

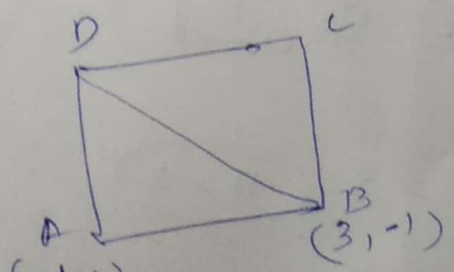
at $x = 45^\circ$ both have f(x) have max

$f(x) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$ $\boxed{B}$

3 Pentagon is divided into Three triangles so

All Angle ~~are~~ sum = $3 \times 180^\circ = 540$

mems is = $\frac{540}{5} = 108^\circ$ $\boxed{C}$

4 

$AB = \sqrt{(3+1)^2 + (1+1)^2} = \sqrt{20}$

$BD = \sqrt{AB^2 + AD^2}$

$= \sqrt{(\sqrt{20})^2 + (\sqrt{20})^2}$

$= \sqrt{40}$ $\boxed{D}$

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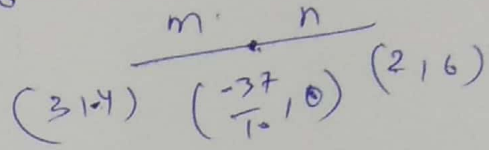
5 Line Joining $(3,-4)$ & $(2,6)$ is

$y-3 = -10(x+4) = -10x-40$

$\Rightarrow$ ~~x+y~~ $10x+40+y-3=0$

$$\boxed{10x+y+37=0}$$

at x, $y=0$, $x = \frac{-37}{10}$



$\therefore 6m-4n=0$

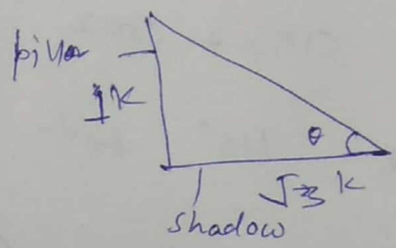
$3m=2n$

$\boxed{\frac{m}{n} = \frac{2}{3}}$ (A)

6

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$ $\boxed{B}$



7 Let five consecutive integers are a, b, c, d, e

products, $abcde = a$

$a(bcde - 1) = 0$

$\because$ product of five integer cannot be 1 so

$a = 0$, $\because$ but max integer

$b > c > d > e - 4$ $\boxed{B}$

8 $2^{x+3} = 32 = 2^5 \Rightarrow x+3=5$ $x=2$

3^{x} = 3^5 = 81 $\boxed{D}$

9 integers between 1 to 100 divisible by 7

$\Rightarrow \frac{100}{7} - \frac{10}{7} = 28 - 1 = 27$ $\boxed{A}$

10 $2x^2 + 7x - 3 = 0$

$\alpha + \beta = \frac{7}{2}$

$\alpha\beta = \frac{-3}{2}$

$(\alpha + 3)(\beta + 3) = \alpha\beta + 3\alpha + 3\beta + 9$

$\Rightarrow -\frac{3}{2} + 3 \times \frac{7}{2} + 9$

$\Rightarrow 9 + \frac{21 - 3}{2} = 18$ $\boxed{D}$

11 $x^3 - x^2 - 4x - 6$, $x^2 - 2x - 3$

factor $= (x-3)(x^2 + 2x + 2)$ $\Rightarrow (x-3)(x+1)$

H.C.F $= (x-3)$ $\boxed{D}$

12 $\frac{1}{\sqrt{6}+\sqrt{5}} + \frac{1}{\sqrt{9}+\sqrt{8}} + \frac{1}{\sqrt{7}+\sqrt{6}} + \frac{1}{\sqrt{8}+\sqrt{7}} + \sqrt{5}$

$\Rightarrow \cancel{\sqrt{6}} - \cancel{\sqrt{5}} + \sqrt{9} - \cancel{\sqrt{8}} + \cancel{\sqrt{7}} - \cancel{\sqrt{6}} + \cancel{\sqrt{8}} - \cancel{\sqrt{7}} + \cancel{\sqrt{5}}$

$\Rightarrow \sqrt{9} = 3$ $\boxed{C}$

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13 $a, b \in R$;: a^2+b^2

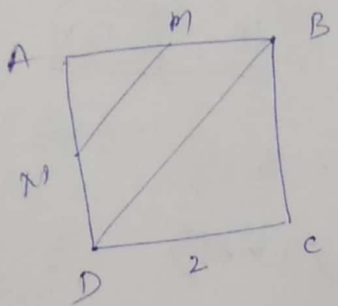
$a > 0$
$b > 0$

$\therefore$ (a^2+b^2) will be tends to zero (A)

14
$21 - 4 = 17$
$38 - 4 = 34$
$55 - 4 = 51$
$108 - 4 = 102$

$\boxed{A}$

15



Area of ABCD $= 2^2 = 4$
Area $\Delta BCD = \frac{1}{2} \times 2 \times 2 = 2$
Area $\Delta AMN = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$
Area. MNDB $= 4 - 2 - \frac{1}{2}$
$= \frac{3}{2} = 1.5$ $\boxed{B}$

16 total/inside sum of a cube = 6 cm²
Volume is $= \left(\frac{1}{2}\right)^3 = \frac{1}{8}$ unit³ $\boxed{C}$

17 Let area is $\Rightarrow L\ B$
Now New $= 2L \cdot 2B = 4LB$
incr. % Area $= \frac{3LB}{LB} \times 100 = 300$ $\boxed{C}$

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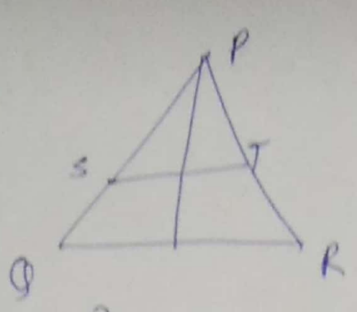
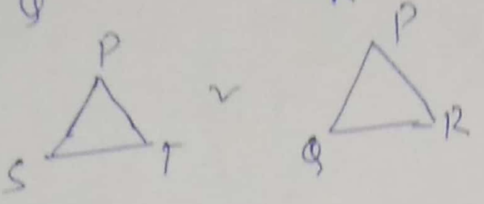
18



ST // RQ

Area △PST = 45 cm²

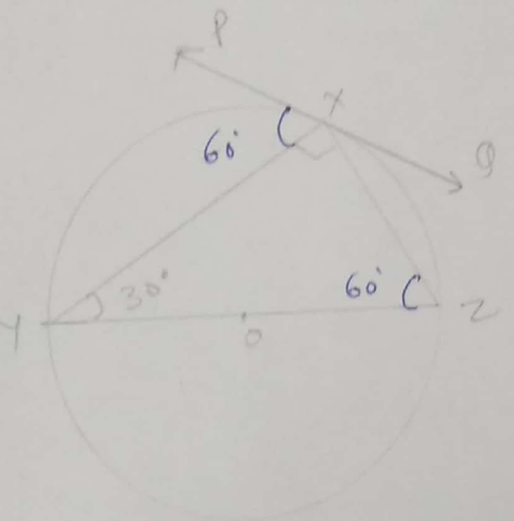
$\frac{PS}{PQ} = \frac{3}{5}$



$\because \frac{\text{Area } \triangle PST}{\triangle PQR} = \left(\frac{PS}{PQ}\right)^2 = \frac{9}{25}$, $A(\triangle PQR) = \frac{9 \times 45}{25} = 125$

Area of STRQ = 125 − 45 = 80 (B)

19

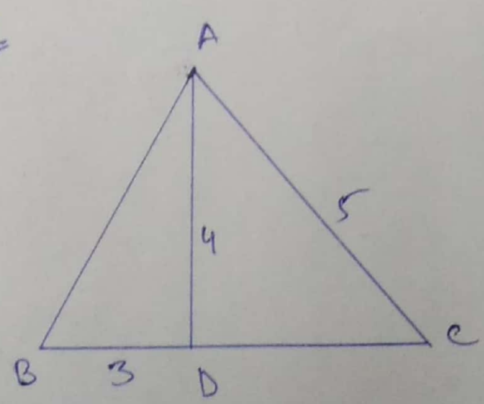


∠PXY = ? = 60° (C)

∠XZY = ∠PXY

60°

20



AD = median

AD = 4, BD = 3, AC = 5

$\frac{1}{2} \times BC \times AD$

$\Rightarrow \frac{1}{2} \times 6 \times 4 = 12 \text{ cm}^2$ $\boxed{(B)}$

Scanned by CamScanner